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Two algorithms to solve non-ionvex geometrical programming problems have been developed and tested. Computational results have been obtained.

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FINAL TECHNICAL REPORT

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Global Minimum Solution of Engineering Design Problems

Many important problems in engineering design lead to a geometric programming formulation:

$$\min_x \varphi(x) = \sum_{i=1}^{n} \alpha_i x_1^{\beta_{i1}} x_2^{\beta_{i2}} \cdots x_m^{\beta_{im}} \tag{1}$$

subject to linear constraints and bounds on the positive design variables $x_j > 0$, $j = 1, \ldots, m$. The exponents β_{ij} are given and may be positive or negative. If all $\alpha_i > 0$, then (1) is convex and easily solved.

If one, or more $\alpha_i < 0$, the problem is nonconvex, and may have many local minima. The design engineer wants to find the global minimum (e.g., minimum cost or minimum weight).

Two different computational methods have been developed for solving problems of this kind.

The first method is a stochastic approach which essentially finds all the local minima by choosing starting points which are uniformly distributed in the feasible space, and the local minimum corresponding to each. A stopping rule is used to determine when all the local minima have been found (with a specified probability).

The second method initially transforms (1) to a separable function. It then gives a guaranteed ϵ-approximate global minimum point, for any user specified tolerance ϵ. It is based on a new theoretical result which gives an easily computed sufficient condition for a global minimum of this type of constrained problem.

Computational results for both methods have been obtained for a range of problems using the Cray X-MP. The stochastic method is very well suited for parallel implementation, and its use on the CM-2 (and possibly the CM-X) is being investigated.

References

[1] A. T. Phillips, J. B. Rosen, M. van Vliet, "A parallel stochastic method for solving linearly constrained concave global minimization problems," UMSI 91/192, July 1991.

[2] A. T. Phillips, J. B. Rosen, "A parallel algorithm for partially separable nonconvex global minimization," *Ann. Op. Res.*, Vol. 25, pp. 101-118, 1990.

[3] J. B. Rosen, A. T. Phillips, "Sufficient conditions for linearly constrained concave minimization: Computational implementation," Presented at 14th Math Programming Symposium, Amsterdam, Aug. 1991.

Solution of Large-Scale Block Structured Problems

Many large-scale constrained optimization problems possess an inherent block structure. In earlier work, supported by this grant, an efficient parallel method for block-angular linear programs was developed and implemented on both the Cray-2 and the NCUBE.

More recent work has extended this parallel method to problems with many nonlinear constraints. This extension (called the RMG method) first solves many relatively small linear programs in parallel (one for each block), and then improves the values of the linking variable by solving a small nonlinear reduced problem. This iterative process is repeated until an optimality test is satisfied.

Computational results using the RMG method, for problems with quadratic constraints, have been obtained on the Cray-2, and a 64-node first generation NCUBE. A range of problems with up to 64 blocks have been solved, the largest consisted of 64 blocks with a total of 3200 variables and 6400 quadratic inequality constraints. This problem took approximately 91 seconds on the Cray-2 and 548 seconds on the 64-node NCUBE.

For comparison, a set of similar problems were solved on the Cray-2, using both MINOS 5.3 and the RMG method. The largest problem solved with MINOS consisted of 16 blocks, 800 variables and 1600 quadratic inequality constraints. It required 576 seconds to solve. The same problem was solved in approximately 27 seconds using the RMG method.

These results have been presented to a group from SSI and IBM, who have expressed an interest in incorporating this method in their mathematical programming software.

References

1. J. B. Rosen, R. S. Maier, "Parallel solution of large-scale block-angular linear programs," *Ann. Op. Res.*, Vol. 22, pp. 23-41, 1990.

2. J. Glick, R. S. Maier, J. B. Rosen, "Parallel solution of large-scale block-diagonal concave maximization problems," To be published in *SIOPT*, 1991.

Multipivot Algorithm for Large-Scale Linear Programs with a Special Structure

An important type of large-scale linear program with a special structure is that with relatively few variables but many inequality constraints. For example, a production problem with m production centers and K possible scenarios will give a problem with m variables and mK inequality constraints. This structure also represents the infinite horizon, discounted Markov decision problem.

A new solution method for this type of problem has been discovered, which is similar to the simplex method but permits multiple pivots at each iteration. This will typically cause a dramatic reduction in the total number of iterations required as compared to the simplex method.

The worst case behavior of this algorithm has been analyzed and it has been shown that no more than 2mK iterations are required.

Computational testing has been carried out using the Cray-2 on a range of problems with m=100 and up to 100,000 inequality constraints. The maximum number of iterations required was 13. The time to solve the largest problem (m=100, mK=100,000) was 6.6 seconds.

Reference

1. J. B. Rosen, S. Oh, "An efficient algorithm for large-scale linear programs with a special structure," *UMSI* 91/145, May 1991.